Guiding Quantum Histories With Intermediate Decomposition of the Identity

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Abstract. The effect of a carefully chosen measurement action is proposed to be able to influence the probability distribution of future outcomes. Using the consistent histories formalism, we may calculate the overlap between a current measurement action and an array of possible future states. We propose an interpretation of the mathematics in which a current measurement action reaches recursively into future states and returns a probability. Using intermediate decomposition of a quantum history, we decompose a state at intermediate times into a complete set of states, using a grouping which distinguishes particular outcomes. The "meaning" of the grouping is defined rigorously, and it is shown that under certain minimal assumptions a 'meaningful grouping' will always increase the likelihood of a particular outcome. This is labeled "meaningful history selection." Grouping histories is not a physical process, but rather an information theoretic one that occurs during the measurement process. The model is consistent with the standard Von Neumann measurement process under normal conditions, but leads to a proposed small deviation in the presence of a conscious observer that naturally accommodates the experimental evidence of certain psi phenomena. It is proposed that the effect of a conscious observer acting on a system is to group the histories in a distinguishable way, thereby minimizing the entropy increase of the system upon measurement. Compatibility with various models of quantum theory are discussed.

INTRODUCTION

From quantum theory (and Bell's theorem in particular) we can surmise that properties of systems do not take definite values separately from measurement of those properties. Mermin [1] says the fundamental point of quantum mechanics is that

> Correlations have physical reality; that which they correlate does not.

Hence the past state of a system should be described by a superposition of *possible histories* (rather than *actual facts*) over a sequence of time steps, obeying the constraints of consistency, completeness and orthogonality. This is the general undertaking of the consistent histories interpretation of quantum mechanics. [2] [3] A history can be broken down using the spectral theorem into any one of many different decompositions with respect to a set of basis states, a process known as "decomposition of the identity."

If a measurement is made of the system at a certain time, a particular intermediate decomposition or basis is selected. The system then collapses to one of the available histories in that decomposition with its probability given by the coefficient of the history.

The overall program in this paper will be to make four new proposals. Firstly we will postulate a robust 'abstract information space' that has objective (but non-physical) characteristics. This is simply the wave function of a system as it branches during interactions. By "objective" in this case we mean it represents potential physical outcomes that can in principle be (indirectly) measured and agreed upon by different Users.

Secondly, when a measurement is made by an observer (we will call it the "User"), the action acts not only on the present state of the system but also projects forward onto the branching tree of Schrödinger evolved states. This is reasonable since kets and bras do not explicitly encode a time parameter: one generally *assumes* (but cannot make rigorous) that the time associated with a measurement operator is the same as the time associated with the entity it measures. By instead noticing that the measurement operator applies against the entire history of the measured entity (forward or backward in time), we are able to use the apparatus of the inner product to determine which future outcomes overlap with the measurement we have made.

Thirdly, we will precisely define the "meaning" of an action in the context of physics. Each branch of the tree (of Schrödinger evolved states) can be assigned a weight based on the inner product described above. The degree to which the branches *differ* in their weights determines the "meaningfulness" of the measurement action. A more meaningful distribution of outcomes is one in which the measurement result is *unevenly* distributed across the branches of the tree and has a *minimal* increase in entropy of distribution.

Finally, we will speculate that the role of a conscious User is to group the histories in a meaningful way, as defined herein. By doing so, the User influences the probability distribution of the set of outcomes, and increases the likelihood of outcomes that align with the User's action.

The proposal that a measurement action projects onto future possible states of the system may meet with skepticism among realists. A realist will wonder how one can discuss "histories" of physical things in a mathematically abstract space. Yet many experimental results in modern physics support this view: instantaneous correlation at a distance [4]; a delayed choice that retroactively determines a prior state of a system [5] [6]; erasing the "fact" that a measurement has been made on a system [7]; separating the properties of a particle from the particle itself [8]. As Mermin alluded to earlier, the 'real' entities are the relationships *between* things, but the things themselves do not have independent existence.

As a result of the above proposals, each branch is assigned actual probabilities based on the projection of the measurement operator onto future possible states. It will be shown that the simple act of taking a measurement action therefore automatically makes 'aligned' measurement outcomes (a time t advanced in the future) more likely.

Furthermore, there is no value judgment placed on events, as one might be concerned about for a theory of "meaning." We will show that by simply acting upon the future possible states of the system, the User's action may be selecting the most "meaningful" outcome of an experiment, based upon the mathematical definition provided. This result is compounded if multiple similar steps are taken in series.

From many carefully controlled experiments that have aimed to understand the possible influence of conscious intention on physical systems, [9] [10] [11] [12] [13] the indication is that although the effect size is small, there exists a robust 'psi' effect with high statistical power that begs explanation. The author wonders whether, given a theory such as the one presented here of "meaningful history selection" that sticks as closely as possible to established properties of quantum theory, a skeptic might be more inclined to seriously consider results such as those listed above. It appears that it is lack of the correct model that continues to hamper acceptance of some of these weak but persistent phenomena.

Compound History Groups vs. Elementary Histories

Before proceeding, it is important to present a little of the material examined in the consistent histories formalism. A history of a quantum system does not have a fundamental level of detail or 'refinement' at which it is the 'correct' description. Rather, one can generally *refine* a history by considering properties of the system that were not previously included in one's description, or alternately *coarsen* a history by ignoring details that were previously considered. Formally, coarsening amounts to summing up the histories which distinguish between the values of a property we wish to ignore, so that the coarsened history provides no information about that property. Refinement, conversely, provides new details about the system with respect to some property that had been previously ignored.

Generally, any system we could consider will have further details that we have ignored, but could potentially choose to consider. By considering these new details of the system, we are expanding into a more refined description which allows us to distinguish between different values of the new property. As a simple illustration, a pen is a single object at a very coarse level description. Our description can be refined by treating the sheath and the ink cartridge as separate objects, so that we are now tracking details that were previously ignored. We can always continue to track more subsystems without ever (practically speaking) getting to the level of tracking every single degree of freedom of the subatomic particles in the pen.

Any grouping of histories is an equally valid description of the system, for one can always coarsen a consistent family of histories by combining histories, and in such a case it remains consistent. In the process of meaningful history selection presented here, we will begin with elementary histories (the highest refinement) and group them according to various criteria in order to obtain compound histories. However, it is important to recognize that compound histories are not approximations. They are in all respects true quantum histories, obeying all laws of quantum systems. If we want to we can always refine them further, but in general we need only refine them to the point where we can track the properties we care about.

With this in mind, the branch of a tree can always be subdivided into a collection of connected smaller branches, should we desire to track further details of the system. The branch itself is not a fundamental structure but a conglomerate of many undifferentiated possibilities.

For our purposes, we will consider both elementary histories and compound histories, although the distinction between them is not fundamental. A compound history is one which can be refined into subhistories, which provide more details about a property of the system. An elementary history is one which cannot be further refined *with respect to a particular property*. We will make use of their slightly different properties in the formalism that follows.

OBJECTIVE VS. SUBJECTIVE MEANING

We begin by introducing the notion of 'objective meaning' into physics. One reason that meaning is considered a part of psychology and not physics is that we fail to distinguish between the psychological phenomena and the fundamental phenomena. Here I propose that we need to distinguish between "subjective" (or psychological) meaning and "objective" (or fundamental) meaning.

The mainstream view is that all meaning is subjective. For instance, imagine I introduce myself to you by saying "Hi, how are you?" However, if the last time somebody said those words to you, they mugged you and stole your wallet, you may have a particular subjective or psychological meaning that is associated with those words. You may react to my harmless question in an unexpectedly defensive manner which is completely unrelated to what is really happening. This is an example of subjective meaning, and has led many to presume that *all* meaning is subjective. It arises from pre-existing psychological conditions, perceptual filters, or networks of assumptions that are true for an individual User, but are not universally true. Meaning of this sort has no fundamental role in physics.

By contrast, we can now introduce a measure of "objective meaning" which is quantifiable and is unrelated to the psychology of the individual. It is labeled "objective" because it is, in principle, measurable with an instrument, rather than based on the relative state of individual observers.

Having discussed the grouping of elementary histories at intermediate times, we are in a position to define "objective meaning" in the following way:

> Objective meaning: A measure of how *unevenly* the identified outcomes are distributed across the branches of the tree in the history space.

This definition is simply a statement about the information contained in the structure of the elementary histories. It is inversely related to the entropy gained by the system as a result of the measurement. A "meaningful action" is one which projects onto future elementary histories that are *not* evenly balanced with respect to a given property of the system, and thereby minimizes entropy of the distribution with respect to that particular property.

A non-meaningful action would therefore be one which results in an even distribution of outcomes that does not distinguish between the various branches of the tree, leading to maximum entropy increase upon measurement and no increase or decrease in the likelihood of that *particular* outcome.

As an aside, while a general quantum measurement can describe interactions between inanimate objects, these do not lead to meaningful groupings. Meaningful groupings are proposed to be a result of some directed action taken, which implies a living creature. We will limit our discussion to measurements made by human Users without investigating the deeper question of carefully defining consciousness, which is a wide field pursued elsewhere. [14] [15] [16] [17] [18]

MEANINGFUL HISTORY SELECTION

We are now at the core of the investigation. A diagram of the process of meaningful history selection is provided in Fig. 1. The proposed elements involved in the process of meaningful history selection are defined as follows:

- History: A chain of quantum states connected in time. A history does *not* only represent past states, but rather can represent states in past, present and future. In particular, we will be projecting the operator representing the User measurement action, defined below, onto the *future outcomes* of each 'history,' where each history is a particular trajectory an entity can follow from past into future.
- Definite state: a state which has been observed by the User.
- $[B]$ and $\langle b|$: User action, an operator determined by the measurement choice of the User. $[B] = |b\rangle\langle b|$ is a projector onto the state $|b\rangle$, and the action can be equivalently represented by either $\langle b|$ or $[B]$.

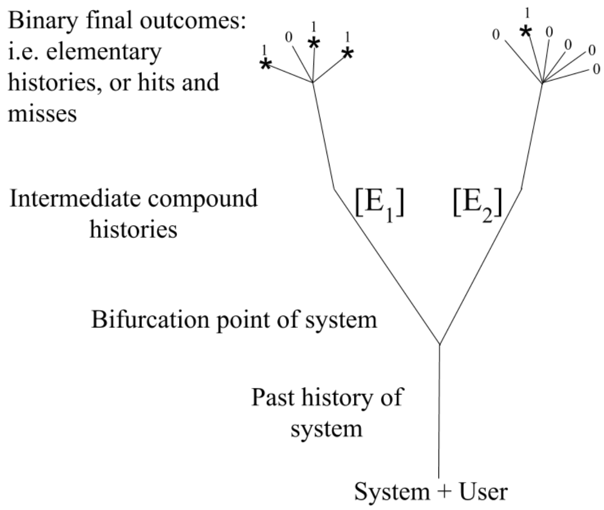

FIGURE 1: The process of meaningful history selection is described by a branching tree, with the reference of time (as an ordering principle) moving up the page. Outcomes having an inner product with the measurement operator equal to unity (i.e. hits) are shown with stars. Branch E_1 will have a heavier weight due to its density of hits, and is therefore more likely to occur. Without weighting the histories, we would expect a $\frac{4}{10}$ = 40% chance of receiving a target outcome. With the weighting, the probability becomes $\frac{3}{4}\frac{3}{4} + \frac{1}{6}\frac{1}{4} \approx 60\%$.[a]

[a] All events are presumed to be space-like; no reference to faster than light signalling is implied in this model. The diagram is not a Minkowski diagram.

- Elementary history, $|e^k\rangle$: a chain of events at a level of refinement that cannot be further refined with respect to a particular property. The likelihood of an elementary history is purely probabilistic and cannot be traced to the properties of its subsystems. An elementary history has binary overlap with the action operator,

$$\langle b|e^k\rangle \approx 0 \text{ or } 1. \tag{1}$$

- Compound history: a chain of events which *can* be further refined into a more detailed description of its subsystems. The likelihood of a particular compound history can be traced to the properties of the elementary histories of which it is composed (i.e. its branches).
- Hits: Elementary histories $|e^k\rangle$ whose inner product with the User action $\langle b|$ are equal to unity, $\langle b|e^k\rangle \approx 1$.
- Misses: Elementary histories $|e^k\rangle$ whose inner product with the User action $\langle b|$ are equal to zero, $\langle b|e^k\rangle \approx 0$.
- Meaningful action: An action whose projection onto future branches of the tree leads to different weights for the nearby branches, thereby distinguishing future outcomes in the history space. A meaningful action results in a distribution of possible outcomes with less than maximal entropy.
- $[E_i]$: A simple intermediate decomposition ("branch") of elementary histories. By convention, if the distribution is uneven with respect to the number of hits, we will take $[E_1]$ as the compound history which contains the higher density of hits, and $[E_2]$ the compound history with the lower number of hits. For simplicity we will assume only two possible branches for the intermediate decomposition, but this is not a requirement.

User Action

The User's action (or "measurement") is represented by a Hermitian operator. Without a specific action, no experiment takes place. The action therefore defines the measurement basis (i.e. the set of possible outcomes). There is not a single fundamental set of 'actual' possible outcomes from an experiment. Rather the choice of measurement basis determines the particular set of possible outcomes available for a particular run of the experiment.

We will consider the action to be a projective measurement of the system, so the operator $[B] = |b\rangle\langle b|$ projects the history space onto the ket $|b\rangle$. Hence we will represent the User's action by $\langle b|$, where it is understood that the action itself is the projector $[B]$, a Hermitian operator. There is a one to one mapping between $[B]$ and $\langle b|$, so there will be no ambiguity.

Projection Onto History Space

The history space can be visualized as a branching tree, where the moment of interaction is represented by the trunk of the tree. The process of projecting with $\langle b|$ onto $[\psi]$ happens recursively onto all future states until a *binary* result is (approximately) reached. We can take the inner product $\langle b|\psi\rangle$ of $\langle b|$ with a given branch, and if the result is not close to zero or unity, we expand the branch into many subhistories (further, smaller branches based on a refinement of the branch) and take the inner product $\langle b|\psi_i\rangle$ of $\langle b|$ with each of these. The subhistories are found from applying the Schrödinger equation to the elements of the system. A branching occurs when two subsystems interact and correlate with each other into distinguishable possible outcomes.

If $|\psi\rangle$ is originally composed of two subsystems $|\phi\rangle$ and $|\eta\rangle$ which have not interacted, then an interaction between the subsystems causes a branching in the tree structure, given by

$$|\psi\rangle = \sum_i \sum_j |\phi_i\rangle \otimes |\eta_j\rangle \rightarrow \sum_i |\phi_i\rangle \otimes |\eta_i\rangle = \sum_i |\psi_i\rangle\,,$$

so that the two systems are now correlated. The inner product $\langle b|\psi_i\rangle$ is a measure of how well the final outcome of a given history overlaps with the action taken. Any branch $|\psi\rangle_i$ for which $\langle b|\psi_i\rangle \approx 0$ or 1 is relabeled $|e^k\rangle$ and recursion stops. The differences between subscript and superscript indices is a matter of clarity but is not significant.

In comparing histories with the action we are checking as to whether the history is consistent with the action. As we get into more detailed branches of the tree, we are looking in more detail at the evolution of the system and its subsystems, so we are more likely to get specific scenarios that are either fully consistent with the action or fully inconsistent with it. If we let the recursiveness run long enough, we land on particular measurement outcomes which give us a "yes/no" answer as to whether that history is consistent with the action. Such an outcome, for which we get $\langle b|e^k\rangle \approx 0$ or 1, is called an elementary history. Generally, it will be convenient to examine the history space recursively until we find these elementary histories, and then use the resulting binary probability amplitudes to perform our statistical calculations. Therefore, for a given elementary history, the intended goal is either met or not met.

As we iterate out many levels through the branches of the tree, the smaller branches tend to carry less statistical weight due to the fact that there are so many other branches created at each step in the recursion. This brings the inner product closer to zero, so as a practical matter our iteration can stop after a finite number of iterations into the future. We can be confident that all histories for which $\langle b|e^k\rangle$ has not converged to unity will eventually converge to zero, and can be treated as approximately zero weight elementary histories.

Meaningful Grouping

When $\langle b|$ occurs, it is hypothesized here that the elementary histories split into groups $[E_i]$, and the probability of each group is calculated. If the division of elementary histories is a meaningful grouping (uneven with respect to hits and misses), $[E_1]$ can be considered the group with the higher hit rate (i.e. it is composed of more elementary histories that match the action criteria), and we assign it a higher weight. The weight of the compound history group is derived from the usual Von Neumann method for projective measurement, [19] and is given by

$$w_1 = P(E_1) = \frac{\sum_k \langle b|e_1^k\rangle \langle e_1^k|b\rangle}{\sum_{k,i} \left|\langle b|e_i^k\rangle\right|^2} = \frac{N_{H1}}{N_{H1} + N_{H2}}, \tag{2}$$

where N_{Hi} is the number of hits in branch i. The weight of a branch is therefore the sum of all hits in a branch over the sum of all hits in both branches. This weight is normalized and can be interpreted as a probability, P.

One could object to our choice of normalization, arguing that we should divide by the total number of both hits and misses, instead of just the hits. Let's consider for a moment that we do this, writing $w_1 = \frac{N_{H1}}{N}$, where N is the total number of branches overall. Because the intermediate decomposition forms a complete basis, the branches represent all possible paths of evolution for the system. As a User, clearly we must actually experience exactly one

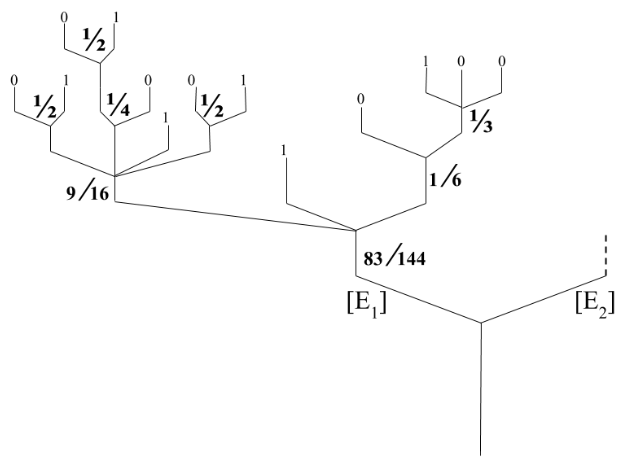

FIGURE 2: A more complex history tree structure, with associated hit counts of elementary histories.

of the branches. Therefore the probability of the branches must sum to unity, and the weights w_1 and w_2 must be normalized. Normalizing the weights $\frac{N_{H1}}{N}$ and $\frac{N_{H2}}{N}$ we obtain

$$P(E_1) = \frac{\frac{N_{H1}}{N}}{\frac{N_{H1}}{N} + \frac{N_{H2}}{N}} = \frac{N_{H1}}{N_{H1} + N_{H2}},$$

which is the same as the expression given above.

To understand this weighting, consider an unlikely event which only has four elementary histories which are hits, and 96 that are misses. If the hits are evenly split between two equal sized history groups, there are two hits out of 50 elementary histories in each group. The likelihood of the event *within* $[E_1]$ remains at $\frac{2}{50} = 4\%$, given by the ratio of hits to total histories. Also, the weight on each group is 50%, because the groups are indistinguishable, with respect to the property in question.

On the other hand, if we split the elementary histories into two equal sized groups, one which contains 3 hits and the other which contains 1 hit, then although the total likelihood of the outcome remains $\frac{4}{100} = 4\%$, the weight of the first history group is $\frac{3}{4}$ and the weight for the second history group is $\frac{1}{4}$. We postulate that this represents the likelihood of that particular group of histories becoming actualized. So even though the event itself is overall very unlikely, the first history group has a bias because it has a greater (although still small) density of hits ($\frac{3}{50} > \frac{1}{50}$). Regardless of the overall likelihood of the particular outcome, our weighting formula allows us to consider how well each compound history group represents the particular outcome, *compared to the other groups*.

Finding weights of history groups through recursion

Equation 2 is valid in a simple scenario where all elementary histories occur at the same level. Specifically, this means that the time evolution of the system is very simple and uniform, so that an initial state evolves symmetrically into an array of final states whose inner products $\langle b|e^k\rangle = 0, 1$. In general, a system is composed of nested subsystems forming a heirarchy of interactions, leading to a tree structure that is quite complex. The elementary histories (where Eqn. 1 holds) then occur at different levels of branching in the tree structure.

In this more general case we must modify Eqn. 2 to account for the *depth of recursion* that we must travel into the tree structure in order to reach a subhistory where Eqn. 1 holds. The contributions of a *compound* history branch at level j are given as the *average* of the contributions of the subhistories of that branch. In other words, we "add up all the leaves on a branch and divide by the number of leaves." This gives us the weight of the branch. We must keep in mind that each 'leaf' is really another smaller branch that can be expanded into subhistories as well, leading to another layer of branches. The process can be truncated at the point when Eqn. 1 holds true for the subhistory. This is a recursion problem well suited to a software algorithm. The weight on the compound history group in the general

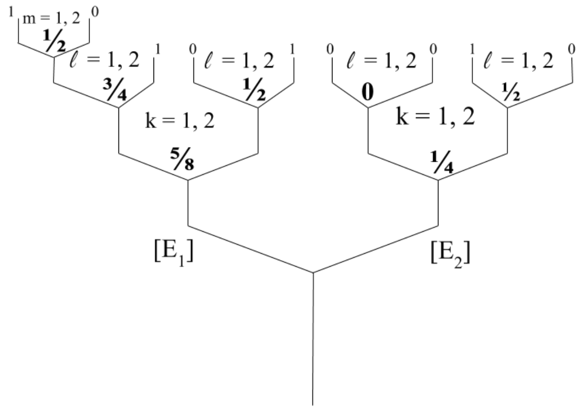

FIGURE 3: The weight of a history group $[E_1]$ can be calculated for elementary histories at arbitrary depths in the tree structure. Here the weight is $P(E_1) = \frac{\frac{5}{8}}{\frac{5}{8}+\frac{1}{4}} = \frac{5}{7}$

case is then given by modifying Eqn. 2 with a recursion relation,

$$w_1 = Pr(E_1) = \frac{N_{H1}}{N_{H1} + N_{H2}} \tag{3}$$

$$\begin{aligned} N_{Hi} &= \sum_j \langle b|e_{ij}\rangle \langle e_{ij}|b\rangle \quad \text{if } \langle b|e_{ij}\rangle = 0, 1 \\ &= \frac{\sum_j \langle b|\mathcal{K}_{ij}|b\rangle}{\sum_j |\langle e_{ij}|e_{ij}\rangle|^2} \qquad \text{otherwise} \end{aligned} \tag{4}$$

where

$$\mathcal{K}_{ij} = \frac{\sum_k \mathcal{P}_{ijk}}{\sum_k |\langle e_{ijk}|e_{ijk}\rangle|^2} \tag{5}$$

and

$$\begin{aligned} \mathcal{P}_{ijk} &= |e_{ijk}\rangle\langle e_{ijk}| \quad \text{if } \langle b|e_{ijk}\rangle = 0, 1 \\ &= \mathcal{K}_{ijk} \qquad \text{otherwise} \end{aligned} \tag{6}$$

where i is the compound history group whose weight we are calculating. Each level of recursion adds a new index to $\mathcal{K}$.

The procedure above allows some subbranches to be elementary histories and other subbranches to be broken up into further subhistories. An example of using this procedure to calculate weights on compound history groups of an arbitrary tree structure is below and shown in Fig. 2.

$$\begin{aligned} N_{H1} &= \frac{\frac{\frac{1}{2}+\frac{\frac{1}{2}+0}{2}+1+\frac{1}{2}}{4} + 1 + \frac{0+\frac{1}{3}}{2}}{3} \\ &= \frac{83}{144} = 0.58. \end{aligned} \tag{7}$$

N_{H2} would be found similarly, and the weight of $[E_1]$ is then given by Eqn. 3. Although an elementary history is defined such that the inner product with the action is either zero or one (Eqn. 1), the hit rate for a given branch can be a number ranging between zero and one, due to the various depths of recursion that must take place to find the elementary histories in the tree structure. In Fig. 3, a simpler example shows how to calculate the final weight on branch $[E_1]$ from the various elementary histories.

Properties of meaningful groupings

The grouping of histories at intermediate times is a meaningless exercise if it does not have physically measurable consequences. In the current quantum framework, although the grouping is perfectly allowable from a theoretical perspective, there is no mechanism by which a User can influence this grouping. Another way to look at this situation is that, all things being equal, nature will tend to divide the histories into groups such that a selection of one state will result in the *maximum possible entropy increase*. This is simply a restatement of the laws of statistical mechanics, where oxygen molecules in a room will never (statistically speaking) group themselves completely in one corner of the room. Similarly, if a system has a large number of possible outcomes (any realistic situation has a very, very large number of them), the vast majority of possible groupings of histories have an equal spread of hits and misses.

For example, consider a system that has $N = 1000$ possible outcomes (this is quite small), where half of the outcomes are hits, $N_H = 500$. If these hits were grouped equally into $[E_1]$ and $[E_2]$, there would be 250 hits in each group. The likelihood of a spontaneous 10% deviation from this can be calculated from the binomial distribution,

$$2 \times \mathrm{CDF}\left[(\mathrm{BinomialDistribution}\left(500, \frac{1}{2}\right), 225\right] = 3\%, \tag{8}$$

where we are calculating the area under the curve of a binomial distribution that arises from finding the chance of getting, say, 225 heads out of 500 flips of a coin, each with 50% probability. The factor of two comes from including the area under both tails of the curve, because we are concerned with cases where we get either 225 or less heads, or cases where we get 275 or more heads.

This is a low estimate for the number of histories available to a realistic system composed of multiple quantum objects over extended time periods. If we increase N by only one order of magnitude, the probability of 10% deviation becomes $2 \times 10^{-10}\%$, which is exceedingly low. In the absence of any new theory, it is therefore overwhelmingly unlikely that one would ever obtain groupings that deviate significantly from the 'even distribution case' through intermediate grouping of events.

One can consider the entropy increase obtained in a measurement which groups the histories in the most statistically likely grouping, where there are an equal number of elementary histories having "hits" in each group (and equal size groups). In order to illustrate, we will consider a system which has 32 elementary histories, eight of which are hits,

$$[E] = 1111\,1111\,0000\,0000\,0000\,0000\,0000\,0000$$

so $P(hit) = \frac{8}{32} = \frac{1}{4}$. Then an even distribution of hits into each compound history would be

$$[E_1] = 1111\,0000\,0000\,0000$$

$$[E_2] = 1111\,0000\,0000\,0000.$$

The normalized weights associated with each history are found from Eqn. 2, which tells us to take a ratio of the number of hits in each history group $[E_i]$ compared to the total number of hits in the sample space of 32 histories. The total number of hits in both groups is $N_{H1} + N_{H2} = 8$, and in the even grouping case the number in each history group $[E_i]$ is $N_{H1} = N_{H2} = 4$. Using the Von Neumann entropy, $S = -\sum_i P_i \ln P_i$, the entropy of this distribution is

$$S = -P(E_1)\ln P(E_1) - P(E_2)\ln P(E_2) = -\frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right) = 0.69. \tag{9}$$

The entropy is maximized when $P(E_1) = P(E_2)$, i.e. when the distribution is uniform. Given the discussion in the previous section, it seems that any grouping that is made spontaneously by nature will maximize the entropy of the outcome measurement (or minimize the information contained in the grouping).

User influence on meaningful groupings of elementary histories

Now let's consider the hypothesis that the user can have some influence on *the way the elementary histories are grouped*. We can examine the Von Neumann entropy increase caused by a "meaningful" grouping of elementary histories, analogous to Eqn. 9. One possible such grouping is

$$\begin{aligned}[E_1] &= 1111\,1000\,0000\,0000 \\ [E_2] &= 1110\,0000\,0000\,0000.\end{aligned} \tag{10}$$

Using Eqn. 2 the weights of these compound history groups are $P(E_1) = \frac{5}{8}$ and $P(E_2) = \frac{3}{8}$. The entropy is

$$S = -P(E_1)\log P(E_1) - P(E_2)\log P(E_2) = -\frac{5}{8}\log(\frac{5}{8}) - \frac{3}{8}\log(\frac{3}{8}) = 0.66,$$

which is less than that obtained in the evenly grouped case. In general any deviation from the random 'natural' distribution will result in a smaller increase in entropy when the measurement occurs. Hence our proposition is that the effect of measurement by a conscious User may be to *minimize the entropy increase* of the system as a result of the measurement. While the second law of thermodynamics ensures that the entropy will always increase in any physical interaction, we postulate that the increase is *minimized* by a conscious observer posing a meaningful question.

It is not altogether surprising that the effect of a conscious User taking action in an experiment may be to minimize the increase in disorder of the resulting system. This is indeed the effect of life in general and consciousness in particular on systems: to increase the local organization of the components of the system.

Calculating the hit probability in a meaningful grouping

It is well known that an even distribution over a discrete sample space gives rise to the maximum value of the entropy. We will assume that all subhistories at a given level in the tree structure are equally likely. Let us now show that *any* grouping of the elementary histories will result in a 'hit' probability greater than or equal to the evenly grouped case.

First we propose that the *overall probability* of a hit is found from Bayesian conditional probability,

$$P(hit) = P(hit|E_1)P(E_1) + P(hit|E_2)P(E_2) = \frac{N_{H1}}{N_1}\frac{N_{H1}}{N_H} + \frac{N_{H2}}{N_2}\frac{N_{H2}}{N_H}, \tag{11}$$

which uses Eqn. 2 with $N_H = N_{H1} + N_{H2}$. This says that the overall probability of a hit is the probability of a hit in $[E_1]$ times the likelihood that we are actually in $[E_1]$ plus the probability of a hit in $[E_2]$ times the likelihood that we are actually in $[E_2]$.

The "evenly grouped" case, where $N_{H1} = N_{H2}$, is only well-defined when $N_1 = N_2$. In this case, $P(hit)_{grouped} = \frac{N_{H1}}{N_1} = \frac{N_H}{N}$. This is equivalent to the probability of getting a hit in the case where no grouping occurs at all, because it is simply the number of hits over the total number of histories. We will therefore use the terms "evenly grouped" and "ungrouped" to mean the same thing. In the case where $N_1 \neq N_2$, the term "evenly grouped" is not well defined, and the term "ungrouped" will strictly be used.

The probability of receiving a hit in the ungrouped case is simply the number of hit outcomes divided by the number of total histories, or

$$P(hit)_{ungrouped} = \frac{P(hit)}{P(hit) + P(miss)} = \frac{\sum_k |\langle b|e^k\rangle|^2}{\sum_k |\langle e^k|e^k\rangle|^2}, \tag{12}$$

where k runs over the total number of elementary histories.

In contrast, the probability of receiving a hit in the grouped case is given by Eqn. 11, where the probability of getting a hit conditional upon being in history $[E_1]$ is

$$P(hit|E_1) = \frac{\sum_k |\langle b|e_1^k\rangle|^2}{\sum_k |\langle e_1^k|e_1^k\rangle|^2}, \tag{13}$$

and the probability of being in history group $[E_1]$ given that we took the action $\langle b|$ is

$$P(E_1) = \frac{\sum_k |\langle b|e_1^k\rangle|^2}{\sum_{k,j} |\langle e_j^k|e_j^k\rangle|^2}, \tag{14}$$

where j runs over the number of groups, and k runs over the number of elementary histories in each group. This results in

$$P(hit)_{grouped} = \frac{\sum_k |\langle b|e_1^k\rangle|^2}{\sum_k |\langle e_1^k|e_1^k\rangle|^2}\frac{\sum_k |\langle b|e_1^k\rangle|^2}{\sum_{k,j} |\langle b|e_j^k\rangle|^2} + \frac{\sum_k |\langle b|e_2^k\rangle|^2}{\sum_k |\langle e_2^k|e_2^k\rangle|^2}\frac{\sum_k |\langle b|e_2^k\rangle|^2}{\sum_{k,j} |\langle b|e_j^k\rangle|^2} = \frac{N_{H1}}{N_1}\frac{N_{H1}}{N_H} + \frac{N_{H2}}{N_2}\frac{N_{H2}}{N_H}, \tag{15}$$

where

$$N_a = \sum_k |\langle e_a^k|e_a^k\rangle|^2$$

are the number of total elementary histories in each group (so $N = N_1 + N_2$), and

$$N_{Ha} = \sum_k |\langle b|e_a^k\rangle|^2$$

are the number of hits in each group, so

$$N_H = N_{H1} + N_{H2} = \sum_{k,j} |\langle b|e_j^k\rangle|^2.$$

In the last definition we are summing over all elementary histories in all groups.

Eqn. 15 seems to be the statement that either one or the other of the history groups $[E_1]$ and $[E_2]$ becomes actualized, even though the elementary histories that they contain have outcomes that are still undetermined because they haven't occurred yet. Although the elementary histories are still in superposition, we are actually trimming some branches of the tree of histories, making them inconsistent with known facts and therefore inaccessible to the system. Therefore the weights given by Eqn. 2 are probabilities rather than probability-*amplitudes*. This is the key step that results in a modified probability distribution after the intermediate decomposition.

The grouped case reduces to the ungrouped case in the limit that $N_{H1} = N_{H2}$ and $N_1 = N_2$, which is the case of equal grouping where the number of hits in each group and the number of total elementary histories in each group are equal.

We can now compare the grouped case to the ungrouped case. We want to show that $P(hit)_{grouped} \geq P(hit)_{ungrouped}$ for any choice of N_{H1}, N_{H2}, N_1, N_2. Comparing Eqn. 12 to Eqn. 15,

$$\frac{\sum_k |\langle b|e_1^k\rangle|^2}{\sum_k |\langle e_1^k|e_1^k\rangle|^2}\frac{\sum_k |\langle b|e_1^k\rangle|^2}{\sum_{k,j} |\langle b|e_j^k\rangle|^2} + \frac{\sum_k |\langle b|e_2^k\rangle|^2}{\sum_k |\langle e_2^k|e_2^k\rangle|^2}\frac{\sum_k |\langle b|e_2^k\rangle|^2}{\sum_{k,j} |\langle b|e_j^k\rangle|^2} \geq \frac{\sum_{k,j} |\langle b|e_j^k\rangle|^2}{\sum_k |\langle e_j^k|e_j^k\rangle|^2} \tag{16}$$

or more simply

$$\frac{N_{H1}}{N_1}\frac{N_{H1}}{N_H} + \frac{N_{H2}}{N_2}\frac{N_{H2}}{N_H} \geq \frac{N_{H1} + N_{H2}}{N}. \tag{17}$$

We therefore need to show that

$$\frac{N_{H1}^2}{N_1} + \frac{N_{H2}^2}{N_2} \geq \frac{N_H^2}{N}, \tag{18}$$

where we know that

$$N_{H1} + N_{H2} = N_H \qquad N_1 + N_2 = N \qquad N_{H1} < N_H \qquad N_{H2} < N_H \qquad N_1 < N \qquad N_2 < N \qquad N_H \leq N. \tag{19}$$

Rearranging and expanding the right hand side of Eqn. 18 gives

$$\left(\frac{N}{N_1} - 1\right)N_{H1}^2 + \left(\frac{N}{N_2} - 1\right)N_{H2}^2 \geq 2N_{H1}N_{H2}.$$

We can simplify $\left(\frac{N}{N_1} - 1\right) = \frac{N_2}{N_1}$ and $\left(\frac{N}{N_2} - 1\right) = \frac{N_1}{N_2}$, so

$$\frac{N_2}{N_1}\frac{N_{H1}}{N_{H2}} + \frac{N_1}{N_2}\frac{N_{H2}}{N_{H1}} \geq 2.$$

Defining $X \equiv \frac{N_2}{N_1}\frac{N_{H1}}{N_{H2}}$, this reads

$$X + \frac{1}{X} \geq 2. \tag{20}$$

We can check where this is minimum by setting the derivative equal to zero,

$$Y' = \frac{d}{dX}\left(X + \frac{1}{X}\right) = 1 - \frac{1}{X^2} = 0,$$

so the extrema are at $X = \pm 1$. Equation 20 is valid only for the positive root, $X = 1$. Since all the input parameters in Eqn. 18 are positive numbers representing counts of histories, we are guaranteed $X > 0$, and this requirement is satisfied. At $X = 1$ the inequality 20 is saturated, giving the minimum value of 2, as required. This is shown in Fig. 4. Given the constraints in Eqns. 19, $X = 1$ occurs when $N_2 = N_1$ and $N_{H1} = N_{H2}$. This is the criteria for even grouping, so we have shown that the meaningful grouping always has a greater probability of hits (Eqn. 15) than a non-meaningful grouping.

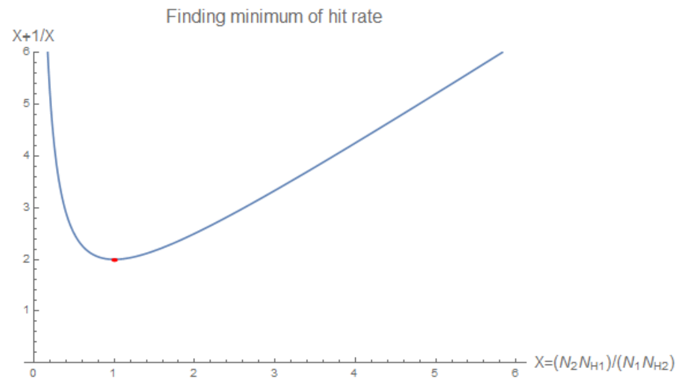

FIGURE 4: Any grouping other than $N_1 = N_2$ and $N_{H1} = N_{H2}$ results in an increased likelihood of hits.

Retroactive Event Determination

We will now discuss how quantum systems remain superposed and available to collapse to a given meaningfully selected state.

We begin with a system defined only at two times, before and after. We can always find out details of the system at some intermediate time by performing unitary evolution of the system, for instance starting from the last event and working backwards to determine which states are possible at the intermediate time.

A simple two-time family of histories can be written

$$[N_1] \odot [\psi_0]$$

$$[N_2] \odot [\psi_0],$$

where $[\psi_0]$ represents the prior history of the system S at time t^1 and the $[N_i]$ represent the two possible outcomes at time t^3. (The initial state is on the right and final state is on the left.)

Next we unitarily extend the system backwards *from* the final state *to* an intermediate time t^2, between t^1 and t^3. Using the projectors $[E_i]$ to represent the states at that time,

$$[N_1] \odot [E_1] \odot [\psi_0]$$

$$[N_2] \odot [E_2] \odot [\psi_0].$$

The time t^2 can be any time between t^1 and t^3, and does not represent a *particular* time. Hence, the span between t^1 and t^3 can in principle be arbitrarily long. There is no fundamental limit on this time span.

We can make a measurement of the system at time t^3. What happens then? We expect to measure either $[N_1]$ or $[N_2]$, but not both. Consequently, the system *is* in one state, and not the other. Experimental data from experiments testing Bell's theorem confirms that the properties measured at t^3 *do not have definite values before* t^3. This is true not only of the state $[N_i]$ at t^3, but also of the backwards unitarily extended state $[E_i]$ at t^2. The properties of the system at t^2 only take actual values at t^3.

This is related to Wheeler's delayed choice gedanken experiment, [20] and has been experimentally demonstrated. [4] [5] It is important to note that this is not a quirk of a particular experiment, but a fundamental principle in quantum systems that is true in general situations. Whenever we perform a projective measurement on a system in a quantum superposition, we get a definite outcome, and the entire history since the system became superposed falls into place when the measurement outcome is obtained.

The name "retroactive event determination" emphasizes that it is not retro*causation*. The properties being measured did not have a definite value prior to the measurement, even in the past. Therefore a prior state cannot be said to be *caused* to take a definite state based on the measurement outcome in the present, but rather its prior state becomes *determined* by the measurement. If a quantum 'fact' has *not* been observed, then it is not a fixed fact, and hence we are not 'changing it.'

One might argue that another User could observe the system in question at the prior time, hence causing the possibility of receiving conflicting stories from the two Users. However, in relational quantum mechanics any interaction between User 1 and the System simply correlates User 1 to the System from the perspective of the User 2. Therefore whatever measurement result User 2 obtains will also retroactively determine the state of User 1. Through the joint principles of relationality and consistency, paradox is neatly avoided. These issues have been discussed in greater depth in previous articles. [21] [22]

Summary of Meaningful History Selection

The process of meaningful history selection is summarized below.

The User takes an action $\langle b|$ by interacting with the system. The action projects onto the history space (tree structure), calculating the inner product with each branch of future possible outcomes. In general, a branch will not have a clear result, in that there are some subhistories of the branch that will result in the User measurement obtaining "Yes" and others such that the User measurement obtains "No." The action then recursively checks each subhistory on that branch, until finding outcomes that answer the question definitively, so that Eqn. 1 is satisfied. The weights on each non-elementary or compound branch can be calculated through the recursive procedure outlined in Eqns. 3-6.

We next introduced the notion of a meaningful grouping of histories. Histories are grouped in a meaningful manner if the grouping tends to increase the information gained by a measurement, or equivalently tends to reduce the entropy of the distribution. We postulate that as a result of an action taken by a conscious User, the groups will each tend to have a different number of hits, resulting in a different weight, so that one group can be distinguished from another based on the property in question.

By convention we choose $[E_1]$ to represent the grouping of histories with the higher number of hits, thus leading to a greater weight w_1 (given by Eqn. 2, or more generally Eqns. 3 through 5) on that compound history. The weight on each compound history group is interpreted as a probability of occurrence for that compound history group.

As the User takes repeated actions, they make certain outcomes impossible and others definite. We can picture this as the tree of available histories having its branches trimmed.

Importantly, we showed that if one accepts this model, the simple acts of identifying and grouping the elementary histories in a meaningful way with respect to property A increases the likelihood of the specified outcome of property A.

Comparison to Formulations of Quantum Mechanics

The approach taken here is consistent with the conceptual and mathematical formulation of quantum mechanics provided by Feynman. [23] The double-slit experiment, for instance, can be seen as a system with many possible physical paths, each path going through one slit or the other. In the language used here, we treat the elementary histories as those paths which take a particular path through the slits. The sum-over-histories approach reflects this same concept when it considers the contribution to the action from every possible path that the particle can take through the slits. The additional proposition in this paper is that a conscious User has the effect of grouping the particular paths in a manner which minimizes the entropy of the grouping. This connection needs to be more fully explored.

In the weak measurement model, [24] one typically relies on the two-state vector formalism, in which both a past cause and a future effect act together to bring an intermediate measurement result. Inherent to this process is a slicing approach, where one divides up the individual outcomes based upon their alignment with a final measurement outcome. Information about the system can be inferred or confirmed via the appropriate slicing of the data, without having actually collapsed the system into an eigenstate during the experiment. There appears to be crossover with the method presented here, and the connection should be explored further.

The relational model of quantum mechanics, as first presented by Rovelli, [25] is a necessary piece of the picture presented here. It is the relational nature of things that allows them to escape objective collapse by decoherence. In the relational model, decoherence is a process occurring *between subsystems*, rather than a process occurring with an "objective environment." For example, consider a computer that measures the radioactive decay of a particle with some probability. Although the information about the state of the decay is stored on internal registers of the computer, which are macroscopic entities that have decohered with the environment, this decoherence is really just correlation with the other entities in the *immediate* environment, or wherever that information has propagated. So long as no relevant information transfer has taken place between the User and the register or its correlated environment, the register remains in a superposition state *with respect to* the User. This is essential for discussing the projection of a User action onto the future state of a system. Without relational quantum mechanics one can only obtain meaningful history selection with systems that are isolated from the *environment*, whereas in the relational view any system obeys meaningful history selection so long as it is isolated from the *User*, which is a vastly less stringent requirement.

The Ithaca Interpretation of Quantum Mechanics [1] is perhaps less well-known, formulated by N. David Mermin, from which the starting quote was taken. This view emphasizes the experimental findings in Bell-type experiments that make it clear that there is no fundamental ground of *things*, but only *relationships among things*. The relational model and IIQM have much overlap. The model of meaningful history selection presented here takes very seriously the propositions from the IIQM, which makes possible the process of retroactive event determination outlined above.

The transactional interpretation shares a similarity to the approach taken here. In the TIQM the forward in time 'retarded' traveling wave function interacts with a backward in time 'advanced' traveling wave, forming a handshake in the intermediate time (the present). The intermediate decomposition of the identity discussed here may be identical to this process. Although the TIQM formulation has an explicit reference to time as a variable in the description of advanced and retarded waves, these details are not included in the consistent histories perspective used in the present paper. Whether a linear metric of time is necessary as a matter of ontological understanding of reality is uncertain. In the approach taken here, we side with consistent histories and rely on time as an ordering principle (e.g. "which event came first?"), without concerning ourselves to its actual value as a metric ("at exactly what time did this event happen?").

The consistent histories or decoherent histories approach [2] serves as a mathematical model for the formalism presented here. This model shows us how to handle a chain of quantum events over an extended period of time. Although we typically *associate* a time index with a ket vector, there is no fundamental place for time to be tracked within a ket. This is not an accident; it reminds us that the value of time is not a fundamental measure in quantum mechanics. Consistent histories embodies this by providing a mathematical structure through which we can project a measurement at some present moment onto the state of a system represented an arbitrary amount of time in the future.

Through the existing formalism it is clear that we can project onto the past states of the system. In the model presented here, we extend the interpretation of the consistent histories formalism to the projection of a measurement action *forward* in time as well. This emphasizes the symmetric nature of the mathematics in consistent histories.

Modeling Types of Psi Phenomena

In the field of psi research there is significant evidence that consciousness can have a small but robust influence on the outcome of physical experiments, as discussed already. This influence appears to diminish over time [10] and be affected by the expectation of the User. [9] [11] It has been suggested [26] that one of the reasons that even well designed experiments to test psi phenomena have difficulty consistently replicating results is that those experiments have not properly controlled for the expectation of the individual running the experiment. The meaningful history selection model has the potential of allowing us to understand these phenomena within a minimally modified quantum framework.

Three well-studied areas of psi that this model may apply to are psychokinesis (PK), precognition, and the experimenter effect. In each field evidence exists indicating a small but robust effect, which is consistent with the model presented here.

Psychokinesis

This refers to a setup that utilizes the bits of a random number generator as the physical system being affected by the User's intended measurement outcome, such as in [27] [13] [12] [28].

The effect of the User action is to project a specific outcome, such as obtaining a 1 bit from the random number generator, onto the future possible outcomes of the system, and group them in a meaningful way according to this property. As a result, the likelihood of the particular outcome increases as described in this paper, and over a large number of repeated trials the User scores above chance in their ability to 'influence' the outcomes of the device.

Psychokinesis is therefore *not* the act of moving physical objects with one's intention, but rather grouping the histories of that object in a meaningful way, such that they are *retroactively determined* in accordance with the intention that was defined in the User action.

Precognition

In some precognitive experiments, a User's biological systems are monitored to determine whether the User responds to an emotionally charged picture just prior to the picture being presented. [29] [30] [31] In the MHS model, as a User continually acts on a system they increase the likelihood of a given outcome by a small amount. This effect is cumulative over repeated actions as the User proceeds outward onto the outer branches of the "tree," where there is a very high density of hits. This means the User is much more likely than before to experience the target outcome. Given that the space of possibilities represents a real mathematical space with an objective structure, it may be reasonable to suppose that such a high hit density proximity might be perceivable by the User.

The experimenter effect

The possible effect of the experimenter's internal bias during an experiment which involves a highly probabilistic, complex or sensitive situation under study is consistent with the model presented here. We have postulated that a User can influence the grouping of histories according to a specific property, as defined by the User's action. If a User believes an experiment will not work, it does not escape our notice that the bias could affect the action $\langle b|$. The User's preference for a given outcome would be projected onto all the elementary histories, and those that confirm the bias would be considered hits. Therefore, according to the proposed model, these outcomes would become slightly more likely, thus confirming the preference of the experimenter.

DISCUSSION

The meaningful history selection model proposes that a User's action projects onto the future states of the system and identifies outcomes which reinforce the User action. The User actually *ends up in* one of these branches, while the other groups of histories become inaccessible. Some potential configurations of the system will therefore never occur, and others become more probable.

One has in mind the image of a branching dendritic tree structure. The Schrödinger equation combined with interactions between subsystems (von Neumann's Process 1 and Process 2) give rise to a vast branching structure of possible histories as the various elements in the measured environment evolve, interact and form correlations. These are the branches of the metaphorical tree, each branch representing a unique configuration of the physical space. When the User takes an action that is consistent with only one of the compound history groups (e.g. $[E_1]$), they are *trimming the tree* to remove any elementary histories associated with the other group (e.g. $[E_2]$). By removing a branch and all of its subbranches from the tree, the density of 'hits' on the remaining tree branches will be higher (or lower) than before. The likelihood of ending up in a branch that is a "hit" has then been modified from prior values for that User.

"Objective meaning" is given a precise definition. Meaning is closely related to information: an increase in meaning corresponds to a clarification of the available possibilities. A meaningful action is one that distinguishes branches of the history tree with regard to a particular property of the system. The degree of meaningfulness of the action can be measured by the entropy of the distribution of outcomes on the available branches of the tree, where a meaningful action leads to a minimal distribution entropy.

Within this framework, one can reasonably claim that repeated directed actions will pull a User towards an intended final future outcome. This proposed effect is both small and cumulative. Over time, through recursive action, what was once a very unlikely outcome can become very likely.

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REFERENCES

[1] N. D. Mermin, Am. J. Phys. **66 (9)**, p. 753 (1998).

[2] R. B. Griffiths, *Consistent Quantum Theory* (Cambridge University Press, New York, 2002).
[3] J. J. Halliwell, Annals of the New York Academy of Sciences **755**, p. 726 (1995).
[4] M. Giustina, M. A. M. Versteegh, S. Wengerowsky, J. Handsteiner, A. Hochrainer, K. Phelan, F. Steinlechner, J. Kofler, J. Larsson, C. Abelln, W. Amaya, V. Pruneri, M. W. Mitchell, J. Beyer, T. Gerrits, A. E. Lita, L. K. Shalm, S. W. Nam, T. Scheidl, R. Ursin, B. Wittmann, and A. Zeilinger, Phys. Rev. Lett. **115 (250401)** (2015).
[5] V. Jacques, E. Wu, F. Grosshans, F. Treussart, P. Grangier, A. Aspect, and J. F. Roch, Science **315 (5814)**, p. 966 (2007).
[6] X.-s. Ma, J. Kofler, and A. Zeilinger, Rev. Mod. Phys. **88**, p. 015005Mar (2016).
[7] S. P. Walborn, M. O. Terra Cunha, S. Pádua, and C. H. Monken, Phys. Rev. A **65**, p. 033818Feb (2002).
[8] T. Denkmayr, H. Geppert, S. Sponar, H. Lemmel, A. Matzkin, J. Tollaksen, and Y. Hasegawa, Nat Commun **5 (4492)** (2014).
[9] B. Dunne, R. Nelson, and R. Jahn, The Journal of parapsychology **61 (3)**, p. 471 (1997).
[10] B. Dunne, Y. Dobyns, R. Jahn, and R. Nelson, J. Sci. Expl. **8 (2)**, p. 197 (1994).
[11] D. Radin, Explore The Journal of Science and Healing **4 (1)**, 25–35 (2008).
[12] D. Radin, L. Michel, K. Galdamez, P. Wendland, R. Rickenbach, and A. Delorme., Phys. Essays **25 (2)** (2012).
[13] R. G. Jahn, B. J. Dunne, R. D. Nelson, Y. H. Dobyns, and G. J. Bradish, J. Scientific Exploration **11 (3)**, p. 345 (1997).
[14] A. Damasio, *The feeling of what happens : body and emotion in the making of consciousness* (Harcourt Brace, New York, 1999).
[15] C. Koch, *The quest for consciousness : a neurobiological approach* (Roberts and Co, Denver, Colo, 2004).
[16] N. Block, Behavioral and Brain Sciences **18**, 272–2876 (1995).
[17] G. Tononi, Biol. Bull. **215**, 216–242 (2008).
[18] D. Chalmers, *The conscious mind : in search of a fundamental theory* (Oxford University Press, New York, 1996).
[19] J. von Neumann, *Mathematical foundations of quantum mechanics* (Princeton University Press, Princeton N.J, 1955).
[20] J. A. Wheeler, in *Mathematical Foundations of Quantum Theory*, edited by A. R. Marlow (Academic Press, 1978).
[21] S. Nelson, J. Sci. Expl. **25**, p. 281 (2009).
[22] S. Nelson, "Retroactive event determination and its relativistic roots," in *Quantum Retrocausation II: Theory and Experiment*, AIP Conference Proceedings, edited by D. Sheehan (American Institute of Physics, San Diego, CA, 2011).
[23] R. P. Feynman, Rev. Mod. Phys. **20**, 367–387Apr (1948).
[24] Y. Aharonov, E. Cohen, and A. C. Elitzur, Phys. Rev. A **89**, p. 052105May (2014).
[25] C. Rovelli, Int. J. Theor. Phys. **35 (8)**, p. 1637 (1996).
[26] J. Palmer, in *Foundations of parapsychology: exploring the boundaries of human capability*, edited by H. L. Edge, R. L. Morris, and J. H. Rush (Routledge Kegan Paul, 1986) Chap. 8, p. 220.
[27] D. Radin, *Entangled Minds* (Pocket Books, New York, 2006), pp. 247–260.
[28] H. Schmidt, J. Am. Soc. Psychical R. **70**, p. 267 (1976).
[29] J. Pers. Soc. Psychol. **100 (3)**, p. 407 (2011).
[30] D. Radin and A. Borges, Explore-NY **5**, 200–211 (2009).
[31] D. Radin, C. Vieten, L. Michel, and A. Delorme, Explore-NY **7** (2011).